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Two Equations for Perfect Numbers

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Abstract

Perfect numbers have not been documented as numerically even. This document shows that the current perfect numbers can be compiled from the difference between two base-two numbers. There are two equations that compile these perfect numbers. As noted by previous mathematicians, perfect numbers that are currently known end in either 6 or 28. To compile perfect numbers that end in the numerical number 6 (except for the perfect number, 6, itself), the difference between two base-two numbers is represented by 2*(256^m) - (16^m). To compile perfect numbers that end in the numerical number 28 (except for the perfect number, 28, itself), the difference between two base-two numbers is represented by 2*(64^n) - (8^n). These equations are explained more in detail later on in this document.

I. Introduction

A perfect number is a number in which its positive divisors sum up to the number itself. Perfect numbers are rare. Within the set of perfect numbers, it can be seen as an exponential growth. Since these numbers grow exponentially, there have only been 51 confirmed perfect numbers. The ancient Greeks were the first to define a perfect number. Then Euclid discovered that his formula of 2^(p-1)(2^(p)-1), includes all of the perfect numbers (but not exclusively perfect numbers). With this formula, a perfect number appeared when the p in the equation was a prime number. Not all prime numbers were perfect numbers. The only problem with this equation is that it does not predict whether or not a perfect number will be odd.

II. Methods

Beginning with the first seven perfect numbers, the difference between two base-two numbers creates a perfect number.

Perfect Number:	Difference Between Two Base-Two Numbers:
6	$2^3 - 2^1$
28	$2^5 - 2^2$
496	$2^9 - 2^4$
8128	$2^{13} - 2^6$
33550336	$2^{25} - 2^{12}$
8589869056	$2^{33} - 2^{16}$
137438691328	$2^{37} - 2^{18}$

Table 1: Perfect numbers and their corresponding base-two differences.

Table 1 shows that perfect numbers can be compiled from the difference between two base-two numbers. This pattern does continue with the rest of the perfect numbers. It is harder to show this pattern with larger perfect numbers due to the numerical limits on handheld calculators. Since the perfect numbers above can be determined by the difference between two base-two numbers, a multiplicative factor of 2 can be pulled from the first base-two number for simplification towards two new equations for perfect numbers.

Perfect Number:	Difference Between Two Base-Two Numbers:
6	$2(2^2) - 2^1$
28	$2(2^4) - 2^2$
496	$2(2^8) - 2^4$
8128	$2(2^{12}) - 2^6$
33550336	$2(2^{24}) - 2^{12}$
8589869056	$2(2^{32}) - 2^{16}$
137438691328	$2(2^{36}) - 2^{18}$

Table 2: Factoring out a multiplicative factor from the first base-two number.

The exponents of the base-two numbers grow at an exponential rate. To reduce these rates of growth, the base-two numbers can be represented by a base other than 2. The table below will show a further simplification of what was the difference between two base-two numbers.

Perfect Number:	Difference Between Two Base-Two Numbers:
6	$2(4^1) - 2^1$
28	$2(16^1) - 4^1$
496	$2(16^2) - 16^1$
8128	$2(16^3) - 8^2$
33550336	$2(16^6) - 16^3$
8589869056	$2(16^8) - 16^4$
137438691328	$2(16^9) - 8^6$
2305843008139952128	$2(16^{15}) - 8^{10}$
265845599156...615953842176	$2(16^{30}) - 16^{15}$

Table 3: Simplification of base-two numbers.

Table 3 shows the simplification of the base-two numbers. The table adds two more perfect numbers to show the pattern more clearly. Table 3 also shows two red perfect numbers. These two perfect numbers cannot simplify any further and thus are not a part of the two equations that will be explained in the future due to their lack in pattern. As seen in the table, the second base-two number in each equation is simplified to a base of either 8 or 16. This pattern will be visible in the simplification of the first base-two number as well, but instead of bases 8 or 16, the bases will be larger such as 64 or 256.

Perfect Number:	Difference Between Two Base-Two Numbers:
6	$2(4^1) – 2^1$
28	$2(16^1) – 4^1$
496	$2(256^1) – 16^1$
8128	$2(64^2) – 8^2$
33550336	$2(256^3) – 16^3$
8589869056	$2(256^4) – 16^4$
137438691328	$2(64^6) – 8^6$
2305843008139952128	$2(64^{10}) – 8^{10}$
265845599156...615953842176	$2(256^{15}) – 16^{15}$

Table 4: Final simplification of the base-two numbers.

The table has a pattern that creates two equations. Now, the perfect numbers that end in 28 are of the form 2*(64^n) - (8^n) and the perfect numbers that end in 6 are of the form 2*(256^m) - (16^m) (except for the perfect numbers of 6 and 28). The known perfect numbers follow this pattern. With this pattern, since the perfect numbers are represented by the difference of base-two numbers, they are even. The numerical values of m and n do not have a definitive equation that determines their values as of thus far.

III. Results

To determine perfect numbers, there are two equations that can be used. The equation used to produce perfect numbers that end in a numerical number 6 (except 6 itself) is produced by 2(256^m) -16^m. The second equation used to produce perfect numbers that end in a numerical number 28 (except 28 itself) is produced by 2(64^n) -8^n.

IV. Conclusion

In conclusion, perfect numbers can be determined by two equations. These equations show a new pattern of thinking when it comes to perfect numbers. Using this pattern, it is still undetermined whether perfect numbers can be odd. With this pattern, future research could be conducted to find the exact values that create perfect numbers and, with this, predict future

perfect numbers.